

# Anomalous Gap Reversal of the $3 + 1/3$ and $3 + 1/5$ Fractional Quantum Hall States

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In this work we report the opening of an energy gap at the filling factor  $\nu = 3 + 1/3$ , firmly establishing the ground state as a fractional quantum Hall state. This and other odd-denominator states unexpectedly break particle-hole symmetry. Specifically, we find that the relative magnitudes of the energy gaps of the  $\nu = 3 + 1/3$  and  $3 + 1/5$  states from the upper spin branch are reversed when compared to the  $\nu = 2 + 1/3$  and  $2 + 1/5$  counterpart states in the lower spin branch. Our findings raise the possibility that the former states have a non-conventional origin.

Over the last three decades we have witnessed an on-going exploration of topological phenomena in electronic systems. Topological ground states may arise from either single-particle band structure effects [1, 2] or from emergent many-body effects in strongly interacting systems. One example of the latter is the fractional quantum Hall state (FQHS) at the Landau level filling factor  $\nu = 1/3$  [3], a ground state belonging to the larger class of conventional Laughlin-Jain FQHSs [4, 5].

More recently it was realized that the family of topological ground states may be much richer than previously thought. Of the novel FQHSs the ones supporting non-Abelian quasiparticles have generated the most excitement [6–8]. The FQHS at  $\nu = 5/2$  is believed to be such a non-Abelian state [9]. However, several other FQHSs in the region  $2 < \nu < 4$ , commonly called the second Landau level (SLL), are also thought to be non-Abelian [10–15].

Despite sustained efforts in theory [10–15], the nature of the prominent odd-denominator FQHSs forming in the SLL, such as the ones at  $\nu = 2 + 1/3$  and  $2 + 1/5$ , remains unknown. The FQHSs at  $\nu = 2 + 1/3$  [16–22] admits both non-Abelian candidate states [10, 11] as well as a conventional Laughlin-Jain description [4, 5]. The relatively poor overlap between the exact and numerically obtained wavefunctions [23–28] and the unusual excitations [15] does not provide firm evidence for Laughlin correlations in the  $\nu = 2 + 1/3$  FQHS. A number of recent experiments of the  $\nu = 2 + 1/3$  FQHS, however, found its bulk [21] and edge [29–31] properties consistent with a Laughlin description. The other prominent FQHS at  $\nu = 2 + 1/5$  [19, 20] is generally believed to be of the conventional Laughlin type [25–28], although there is a non-Abelian construction for it as well [11]. It is therefore currently not clear whether or not the prominent odd-denominator FQHSs in the SLL, such as the ones at  $\nu = 2 + 1/3$  and  $2 + 1/5$ , require a description beyond the conventional Laughlin-Jain theory.

Experiments on the odd-denominator FQHS in the SLL have been restricted almost exclusively to the  $2 < \nu < 3$  range, called the lower spin branch of the SLL

(LSB SLL). Motivated by their poor understanding, we have performed transport studies of these FQHSs in the little known upper spin branch of the SLL (USB SLL), i.e. in the  $3 < \nu < 4$  region. We establish a new FQHS at  $\nu = 3 + 1/3$  by detecting the opening of an energy gap. A quantitative comparison of the gap at this and other filling factors reveals two surprising findings: 1) the ground state at  $\nu = 3 + 2/3$ , a symmetry-related filling factor to  $\nu = 3 + 1/3$ , is not a FQHS, despite the existence of a strong depression in the longitudinal magnetoresistance and 2) most intriguingly, the activation energy gaps  $\Delta$  of the prominent odd-denominator FQHSs are reversed across different spin branches of the SLL. Indeed, in stark contrast to the well established relation  $\Delta_{2+1/3} > \Delta_{2+1/5}$  between the gaps of FQHSs of the the LSB SLL, in the USB SLL we find  $\Delta_{3+1/3} < \Delta_{3+1/5}$ . Within the conventional Laughlin-Jain picture we are unable to account for this anomalous gap reversal. Our result raises therefore the possibility of a non-conventional origin at least for a subset of the FQHSs of the upper spin branch.

In order to thermalize electrons close to their ground state we utilize our ultra-low temperature setup consisting of a He-3 immersion cell [16, 32]. Cooling is ensured by eight sintered silver heat exchangers are immersed in the liquid He-3 bath. Thermometry is performed using a quartz tuning fork viscometer which monitors the temperature dependent viscosity of the He-3 bath [32].

We measured a high quality sample, in which we have already studied transport in the LSB of the SLL [21]. Figure 1 shows this region of the LSB SLL at magnetic fields  $B > 4.1$  T. In this region at  $\nu = f$  we observe a multitude of FQHSs as distinguished by a vanishing longitudinal magnetoresistance  $R_{xx}$  and Hall resistance  $R_{xy}$  quantized to  $h/fe^2$  [3]. We also observe four reentrant integer quantum Hall states (RIQHSs) signaled by quantization of  $R_{xy}$  to an integer, either  $h/2e^2$  or  $h/3e^2$  [33, 34].

Extending measurements to lower  $B$ -fields, we access the USB SLL. As seen in Fig.1, in this region we observe known FQHSs at filling factors  $\nu = 7/2$ ,  $3 + 1/5$ ,  $3 + 4/5$  [33] and four RIQHSs [33, 34]. These FQHSs and

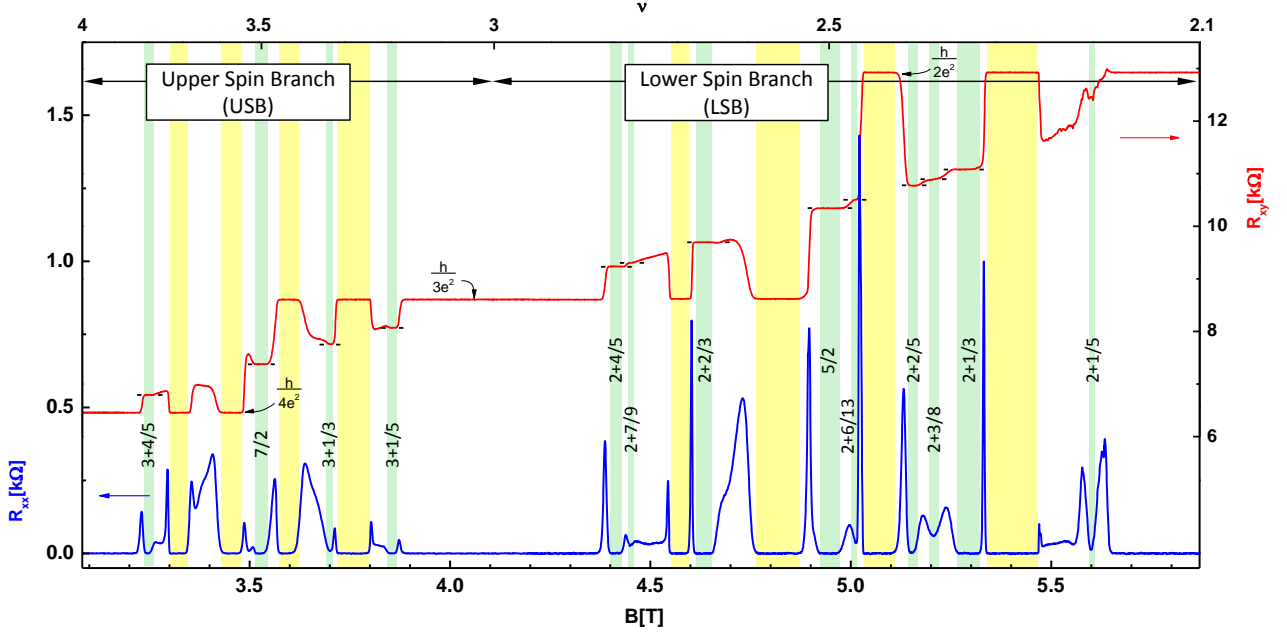


FIG. 1. Magnetoresistance traces in the second Landau level, i.e. in the filling factor range  $2 < \nu < 4$ , measured at  $T = 6.9$  mK. The region of the lower spin branch (LSB) and upper spin branch (USB) are clearly marked. Fractional quantum Hall states are shaded in green, while the reentrant integer quantum Hall states in yellow. The overall symmetry between these two spin branches is evident by the development of fractional quantum Hall states and reentrant integer quantum Hall states at similar partial filling factors.

RIQHSs form at the same partial filling factors, defined as decimal part of the filling factor  $\nu$ . The various ground states in the two spin branches are connected by particle-hole symmetry [35], therefore the ground states at  $\nu$ ,  $5 - \nu$ ,  $1 + \nu$ , and  $6 - \nu$  are said to be symmetry-related or conjugated states. For example, the FQHSs shown in Fig.1 at  $\nu = 2 + 1/5$ ,  $2 + 4/5$ ,  $3 + 1/5$ , and  $3 + 4/5$  belonging to the different spin branches are symmetry-related.

As seen in Fig.1, strong local minima in  $R_{xx}$  also develop in the USB SLL at  $\nu = 3 + 1/3$  and  $\nu = 3 + 2/3$ . However, the presence of these minima does not guarantee the formation of a FQHS at these filling factors. It is known that at  $\nu = 1/7$ , for example, no FQHS develops even though a depression in  $R_{xx}$  is present at finite temperatures [36]. A defining feature of a FQHS, and of any topological ground state in general, is the opening of an energy gap in the bulk of the sample. An energy gap  $\Delta$  is signaled by an activated magnetoresistance  $R_{xx}$  with a  $T$ -dependence of the form  $R_{xx} \propto e^{-\Delta/2k_B T}$ . Other hallmark properties of a FQHS are a quantized Hall resistance  $R_{xy}$  and a vanishing  $R_{xx}$  in the limit of  $T = 0$  [3]. While weak indications of FQHSs have been reported at  $\nu = 3 + 1/3$  or  $3 + 2/3$  in Ref.[33], none of the above described hallmark properties of a FQHS have been observed. A close-up of the USB SLL is shown in Fig.2. We can see that at  $\nu = 3 + 1/3$ , our  $T = 6.9$  mK data exhibit both a vanishingly small  $R_{xx}$  as well as an  $R_{xy}$

consistent with a plateau quantized to  $h/(3 + 1/3)e^2$ .

Magnetotransport at  $\nu = 3 + 2/3$ , however, is markedly different from that at  $\nu = 3 + 1/3$ . As seen in Fig.2,  $R_{xx}$  develops a local minimum at  $\nu = 3 + 2/3$ . However, as seen in Fig.2,  $R_{xy}$  at  $\nu = 3 + 2/3$  clearly deviates from the quantum value  $h/(3 + 2/3)e^2$ , the expected value for a FQHS at this filling factor, casting a doubt on whether the ground state at  $\nu = 3 + 2/3$  is a FQHS. Furthermore, as also shown in Fig.2,  $R_{xx}$  at  $\nu = 3 + 2/3$  increases with a decreasing temperature, suggesting that  $R_{xx}$  does not vanish as  $T$  is lowered.

A detailed temperature dependence of the  $\nu = 3 + 1/3$  and  $3 + 2/3$  FQHSs is shown in Fig.3b. Demonstrated by the linear segments in the arrhenius plots shown in Fig.3b,  $R_{xx}$  measured at  $\nu = 3 + 1/3$  is found to be activated. The opening of an energy gap  $\Delta_{3+1/3} = 37$  mK unambiguously establishes, for the first time, the formation of a new FQHS at  $\nu = 3 + 1/3$ . From data shown in Fig.3a and Fig.3b, we extract the energy gaps of the other odd-denominator FQHSs in the SLL:  $\Delta_{3+1/5} = 104$  mK,  $\Delta_{3+4/5} = 113$  mK,  $\Delta_{2+1/5} = 210$  mK, and  $\Delta_{2+4/5} = 212$  mK. Errors due to scatter in the data amount to  $\pm 5\%$ .

Fig.3b also reveals that the  $T$ -dependence at  $\nu = 3 + 2/3$ , in contrast to that at  $\nu = 3 + 1/3$ , is not activated. The FQHS at  $\nu = 3 + 2/3$  thus does not develop an energy gap in our sample in spite of the presence of a local minimum in  $R_{xx}$ . The ground state at  $\nu = 3 + 2/3$

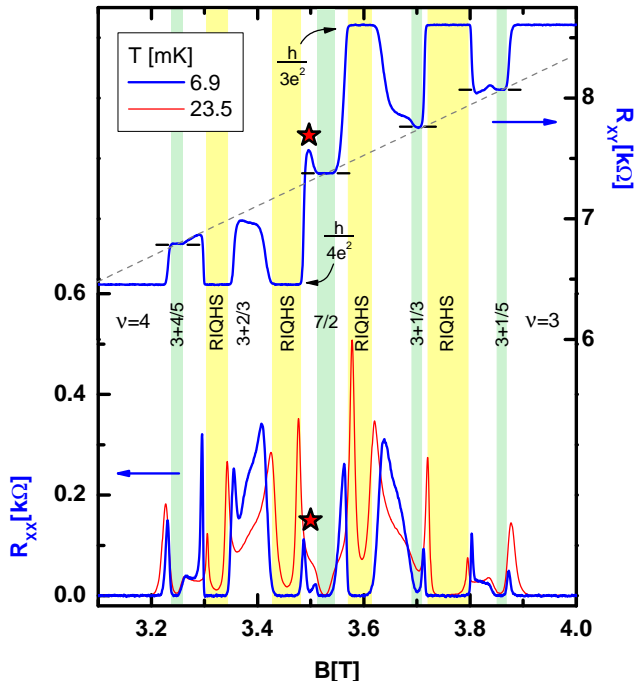


FIG. 2. The magnetoresistance in USB SLL ( $3 < \nu < 4$ ) at 6.9 mK as function of the magnetic field  $B$  (bottom scale) and filling factor (upper scale). Numbers mark various filling factors of interest. We note the absence of a green shading at  $\nu = 3 + 2/3$ , as a FQHS does not develop here, even though a local minimum is present in  $R_{xx}$  at this filling factor. The dashed line is the classical Hall line and the star symbol is indicative of a developing RIQHS of a new type described in the text.

is therefore not a FQHS. However, the emergence of a fractional quantum Hall ground state at this filling factor in future higher quality samples cannot be ruled out at this time.

The energy gaps in the LSB SLL satisfy the  $\Delta_{2+1/3} > \Delta_{2+1/5}$  relationship. This is a well established inequality in many samples of various electron densities [16, 19–22]. Similar inequalities are also known in the lowest Landau level. Indeed,  $\Delta_{1/3} > \Delta_{1/5}$  found in the LSB LLL [37–39]. Furthermore, there is evidence that in the USB LLL the  $\nu = 1 + 1/3$  FQHS is more prominent than the  $\nu = 1 + 1/5$  FQHS [40, 41]. Therefore it appears that the FQHS at partial filling factor  $1/3$  is more stable (i.e. it has a larger energy gap) than that at partial filling  $1/5$ . To our surprise, however, this generally observed relationship is reversed in the USB SLL. Specifically, we find that  $\Delta_{3+1/5} > \Delta_{3+1/3}$ . This anomalous gap reversal in the USB SLL indicates an unanticipated difference between the prominent odd-denominator FQHSs forming in different spin branches.

The anomalous  $\Delta_{3+1/5} > \Delta_{3+1/3}$  gap reversal may be caused by a suppression of the FQHS at  $\nu = 3 + 1/3$

due to a spin transition in this state. Experiments so far have not detected any sign of a spin transition in either the  $\nu = 2 + 1/3$  or the  $2 + 1/5$  FQHSs and NMR measurements at  $\nu = 2 + 1/3$  are consistent with fully spin polarized state [22, 42, 43]. While a spin transition has recently been observed in a related FQHS at  $\nu = 2 + 2/3$  [43], this transition occurs at a magnetic field  $B \sim 1.24$  T considerably lower than the field  $B = 3.7$  T the  $\nu = 3 + 1/3$  FQHS forms in our sample. We thus think spin is not likely to play a significant role in a possible suppression of a FQHS at  $\nu = 3 + 1/3$ .

An anomalous  $\Delta_{3+1/5} > \Delta_{3+1/3}$  gap reversal may also be caused by Landau level mixing [44] or finite width effects [25]. Landau level mixing is a gap reducing effect due to the unoccupied Landau levels above the Fermi energy and its magnitude is enhanced with reduced  $B$ -fields. Similarly, finite width effects change with the  $B$ -field as they scale with the  $w/l_B$  ratio. Here  $w$  is the width of the quantum well and  $l_B = \sqrt{h/eB}$  the magnetic length. In our sample the  $\nu = 3 + 1/3$  and  $3 + 1/5$  FQHSs develop at lower magnetic fields than their symmetry related counterpart FQHSs at  $\nu = 2 + 1/3$  and  $2 + 1/5$  and may be influenced by the two effects discussed above. However, a reversal of the  $\Delta_{2+1/3} > \Delta_{2+1/5}$  inequality has never been detected in any experiments, even when the electron densities are as low as  $n \approx 7.7 \times 10^{10}/\text{cm}^2$  [22, 43]. We thus conclude that the gap reversal of the prominent odd-denominator FQHSs of the SLL is not present in the LSB at any sample conditions, therefore it is an exclusive characteristic of the USB.

With spin and Landau level mixing effects ruled out, we find that the anomalous gap reversal of the  $\nu = 3 + 1/3$  and  $3 + 1/5$  FQHSs cannot be accounted for within the Laughlin-Jain description. One possible cause for this

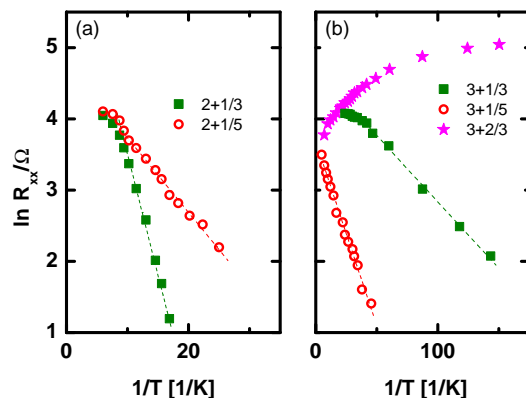


FIG. 3. Arrhenius plots of the  $R_{xx}$  minima at several odd-denominator filling factors in the LSB (panel a.) and USB (panel b.) of the SLL. Data at  $\nu = 2 + 1/3$  is from Ref.[21].

anomalous behavior is the formation of fundamentally different FQHSs in different spin branches. An alternative possibility is that FQHSs at the sample partial filling are the same in different spin branches, but there are fundamental differences between the states with partial filling  $1/3$  and those with  $1/5$ . It is interesting to note that this latter scenario is supported by the results of a recent experiment in which the second electrical subband of a quantum well was populated [45]. In this experiment, populating the second subband had qualitatively different effects on the FQHSs at partial filling  $1/3$  and  $1/5$  in the LSB SLL. It was found that the  $2 + 1/3$  and  $2 + 2/3$  FQHSs became more robust, whereas the  $2 + 1/5$  and  $2 + 4/5$  FQHSs were destroyed [45]. The anomalous gaps we found and the contrasting results reported in Ref.[45] highlight the lacunar understanding of the prominent odd-denominator FQHSs of the SLL and even elicits the provocative possibility that some of the FQHSs may not be a conventional Laughlin-Jain type, but rather of an unknown origin. We note that the possibility that the odd-denominator FQHSs studied here are of non-Laughlin type cannot be ruled out on theoretical grounds. Indeed, as mentioned in the introduction, the overlap of the exact and numerically obtained wavefunctions is not satisfactory for a firm assignment of these states to Laughlin states [23–28] and alternative theories exist which are distinct from the Laughlin-Jain construction [10–14].

At a given filling factor the theory allows the existence of several fundamentally different ground states and even considers transitions induced between these different states. At the root cause of the formation of the various ground states we find minute differences in the effective electron-electron interaction potential caused by changes in the sample parameters. We think that the anomalous gap reversal observed is due to the difference in the effective electron-electron interaction when we populate either the USB SLL or the LSB SLL. We surmise that the study of FQHSs at sample parameters which modify the electron-electron interactions may therefore be of fundamental importance in tuning topological order and may provide a pathway to discovery of novel topological ground states.

Our data reveal that the modified electron-electron interactions in the SLL have another unforeseen consequence. As seen in Fig.2, at the location  $B = 3.50$  T of the star symbol,  $R_{xx}$  is nearly vanishing and  $R_{xy}$  exceeds the classical Hall value. Such a behavior is inconsistent with a FQHS; we think it is a signature of an incipient RIQHS. However, this incipient RIQHS observed at  $B = 3.50$  T, is different from the known RIQHSs [33, 34]. Indeed, the two known RIQHSs at  $\nu > 7/2$ , which develop at  $B = 3.32$  T and  $3.45$  T have  $R_{xy}$  quantized to  $h/4e^2$ , whereas the incipient RIQHS at  $B = 3.50$  T appears to develop towards  $h/3e^2$  in the limit of  $T = 0$ .

In summary, we have probed the upper spin branch

of the second Landau level which appears to be richer than originally thought. Our energy gap measurements of the odd-denominator FQHSs in this region allowed for a test of the symmetry relations between these FQHSs and revealed an unexplained relative magnitudes of these energy gaps. Furthermore, we observed a nascent RIQHS of an unusual type.

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